

NUCLEAR PHASE TRANSITIONS IN TRANSPORT THEORY

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Abstract

Questions concerning nuclear phase transitions are addressed within the transport theory. The issue of thermal equilibrium in the region of the liquid-gas phase transition is investigated. Signatures of the transition to the quark-gluon plasma are looked for.

1 Introduction

Under the idealized conditions of thermal equilibrium, the liquid-gas phase-transition for nuclear matter is a necessity [1]. However, is there enough time in the reactions to reach the equilibrium? The numerical QCD lattice calculations exhibit a phase transition to the quark-gluon plasma (QGP). However, can the excited dense phase leave behind any clear signals in reactions?

We shall try to address the above questions within the transport model based on the Boltzmann equation.

2 Boltzmann Equation

The Boltzmann equation has the same general form relativistically as nonrelativistically:

$$\frac{\partial f}{\partial t} + \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}} \frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{r}} \frac{\partial f}{\partial \mathbf{p}} = \mathcal{K}^{\text{in}} (1 - f) - \mathcal{K}^{\text{out}} f. \quad (1)$$

Here, $\epsilon_{\mathbf{p}}$ is the single-particle energy, $\partial \epsilon_{\mathbf{p}} / \partial \mathbf{p}$ is the velocity, $-\partial \epsilon_{\mathbf{p}} / \partial \mathbf{r}$ is the force, and \mathcal{K}^{in} and \mathcal{K}^{out} are the feeding and removal rates, respectively. The single-particle energy and momentum, $(\epsilon_{\mathbf{p}}, \mathbf{p})$, form a 4-vector. Ensuring covariance in a model can be aided by using a scalar potential U_s dependent on the scalar density $\rho_s = \sum_X A_X \int d\mathbf{p} \frac{m}{\epsilon_p} f_X$. In our model, we also add to the energies

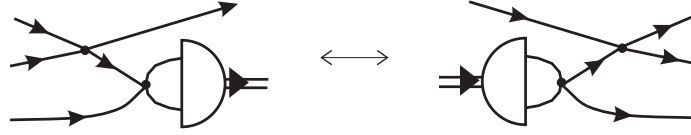


Figure 1: The deuteron production process run backwards in time becomes a breakup process.

a noncovariant contribution accounting for the Coulomb repulsion, isospin dependence, and finite-range effects. The single-particle energies are then

$$\epsilon_p = \sqrt{p^2 + (m^0 + U_s)^2} + U_v, \quad (2)$$

with

$$U_s = \frac{-a\xi + b\xi^\nu}{1 + (\xi/2.5)^{\nu-1}}, \quad (3)$$

$\xi = \rho_s/\rho_0$, and

$$U_v = V_{\text{coul}} + d\nabla^2\rho + ct_3\rho_T \quad (4)$$

where the isospin density is $\rho_T = \sum_X t_3^X \rho_X$.

Besides nucleons and mesons, we consider in our model the lightest clusters ($A \leq 3$) whose phase-space distribution functions follow similar transport equations to those for the other particles. An obvious issue with the clusters is that of their production. Beyond constituents, a minimum of one additional nucleon must be involved in the process, as a catalyst. Let us take the deuteron as an example. In the kinetic limit of low rates, the rate for deuteron production in three-nucleon collisions is given by the the matrix element for production squared convoluted with the δ -functions for the energy-momentum conservation and product of statistical factors:

$$\begin{aligned} \mathcal{K}^{\text{in}}(\mathbf{p}) &= \sum_{N=n,p} \int d\mathbf{p}_N d\mathbf{p}_p d\mathbf{p}_n d\mathbf{p}'_N \overline{|\mathcal{M}^{npN \rightarrow Nd}|^2} \\ &\times \delta(\mathbf{p} + \mathbf{p}_N - \mathbf{p}_p - \mathbf{p}_n - \mathbf{p}'_N) \\ &\times \delta(\epsilon_d + \epsilon_N - \epsilon_p - \epsilon_n - \epsilon'_N) f_p f_n f'_N (1 - f_N). \end{aligned} \quad (5)$$

This apparently just transfers the problem of production to the issue of determining the matrix element for production. Because of the time-reversal invariance, though, the production process run backwards in time becomes the breakup process, as illustrated in Fig. 1, and, correspondingly, the two processes share the matrix element squared:

$$\overline{|\mathcal{M}^{npN \rightarrow Nd}|^2} = \overline{|\mathcal{M}^{Nd \rightarrow Nnp}|^2}. \quad (6)$$

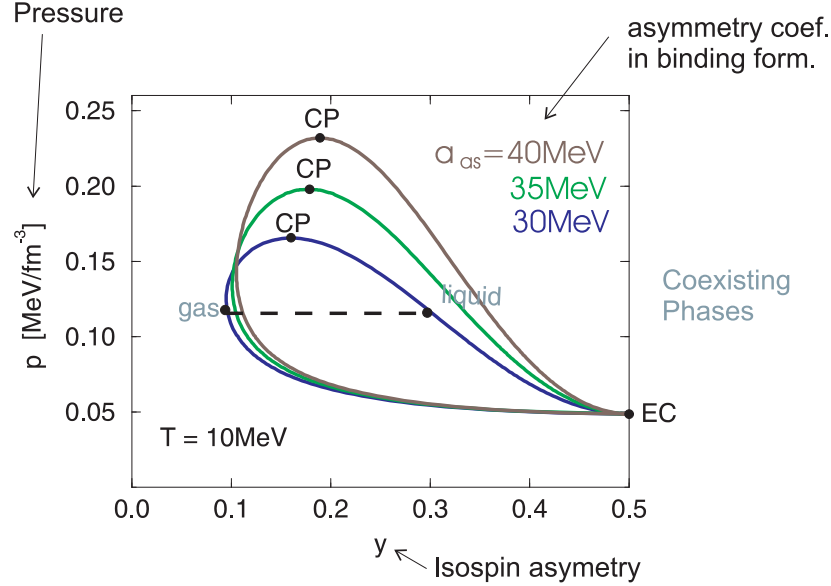


Figure 2: Liquid-gas phase-coexistence in the plane of pressure and isospin asymmetry at $T = 10$ MeV, for different parametrizations of interactions in nuclear matter, after Ref. [3].

The squared element for the breakup is proportional to the breakup cross section

$$|\overline{\mathcal{M}^{Nd \rightarrow Nnp}}|^2 \propto \frac{d\sigma_{Nd \rightarrow Nnp}}{d\mathbf{p} d\Omega}, \quad (7)$$

and, thus, the data on breakup can be used to describe production. The production of $A = 3$ clusters can be handled in a similar manner [2].

3 Isospin Asymmetry in Low-Density Nuclear Matter

Under equilibrium, in the phase-transition region for isospin-asymmetric nuclear, the asymmetry is decreased in the denser liquid phase. This is both because the interactions play a greater role in the liquid than in the gas phase, and they favor the symmetry, and because of the Pauli principle. The enhanced symmetry leaves a pronounced asymmetry in the gas phase. This was recently discussed by Müller and Serot [3], and earlier by Barranco and Buchler [4], and by Glendenning [5], and is illustrated in Fig. 2. The difference in asymmetry may be as large as by a factor of 3. An obvious issue is whether pronounced asymmetries can be seen in reactions and used as a signature for the phase coexistence.

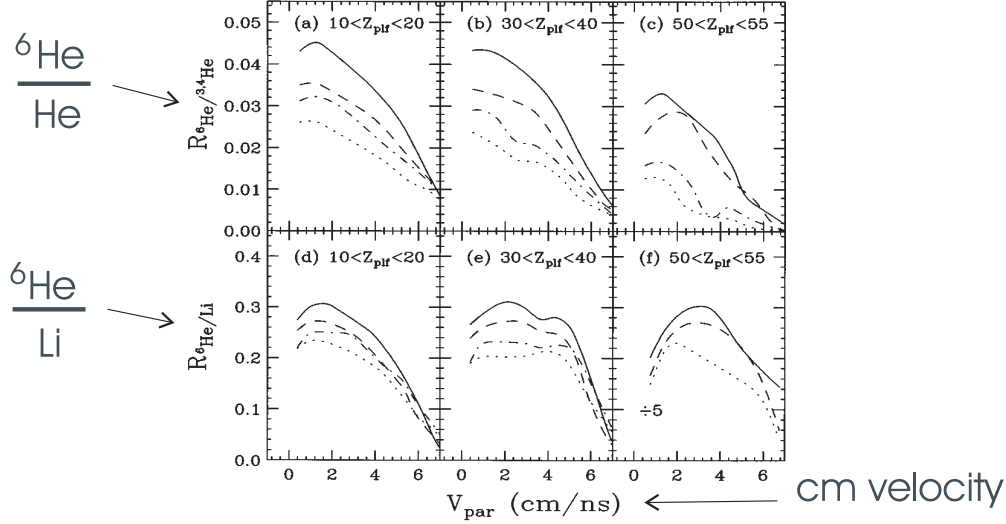


Figure 3: Relative isotope ratios in Xe-Sn collisions as a function of the velocity along the beam in cm [6]. Different lines represent results from different isotopic Xe-Sn combinations.

The problem with the gas phase is in its low density. If anything corresponding to that phase is emitted, it will most likely just escape to the vacuum. The seemingly only practical way to trap the phase and reach any kind of state of phase coexistence is in the neck region between two slow nuclei, with the matter in the nuclei playing the role of a liquid phase. This has a further advantage that the gas phase is then well localized in the velocity space. Dempsey *et al.* [6] studied relative yields of different isotopes emitted from xenon-tin reactions at 55 MeV/nucleon and found large enhancement in the relative abundance of neutron-rich fragments towards midrapidity, as illustrated in Fig. 3.

To verify whether the observed enhancements might signal effects of the phase equilibrium, calculations of the reactions have been carried out [7] within the transport model with isospin asymmetry in the interactions and in the Pauli principle, and with the inclusion of light clusters, as discussed before. Figure 4 shows the contour plots for nucleon, proton, and neutron density, projected onto the plane of the $^{136}\text{Xe} + ^{124}\text{Sn}$ reaction, at different times, and the neutron-to-proton ratio along the axis joining the two nuclei. The ratio is shown both including the protons and neutrons bound in clusters and excluding them. When the simulation is carried without clusters, the results for the ratio are similar to those for *all* protons and neutrons. It is seen that

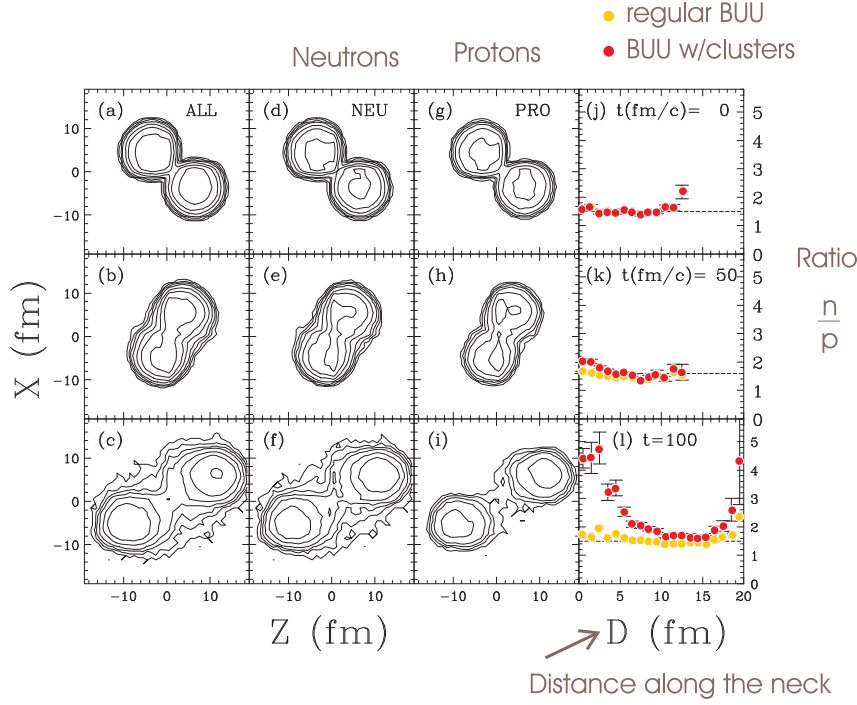


Figure 4: Contour plots for nucleon, proton, and neutron density projected onto the plane of $^{136}\text{Xe} + ^{124}\text{Sn}$ reaction, at different times, and the neutron-to-proton ratio along the axis joining the two nuclei [7].

the ratio for all nucleons just slightly exceeds the N/Z ratio of 1.5 for the whole system. When the clusters are excluded, the ratio reaches high values, up to 4.

The reason for the enhancement in the calculation is the deuteron formation which robs the remainder of the neck region of protons enhancing the asymmetry for the remainder. One can expect that, in reality, alpha particles play an analogous role to deuterons leading to the asymmetries observed experimentally. On the other hand, the calculations show that the overall isospin asymmetry between the gas and liquid phases, due to the favoring of the symmetry in the liquid and expected in the equilibrium, has no time to develop. This brings in the general issue of equilibration within the neck region.

The isospin ratio may be viewed as a convenient variable to assess the equilibration within the phase transition region that, seemingly is not reached. Calculationally, it is far easier to determine the isospin asymmetry than e.g. pressure or temperature. On the other hand, one can argue that the isospin equilibrates in a slow diffusive process (as e.g. smell, aided, though, in the air by convection) while such quantities as pressure or overall density equilibrate

rapidly in a mechanical process mediated pressure waves. Thus, isospin could not be a reliable measure.

Let us take L as a distance over which adjustments of a thermodynamic quantity are to propagate and t as time that the propagation takes. In a diffusive process, a particle undergoing N collisions covers a typical distance:

$$L^2 = N \lambda^2, \quad (8)$$

where λ is the mean-free-path. The collision number is related to the net time by $t = N \lambda/v$, where v is a typical particle speed. The characteristic time for the isospin adjustments is then:

$$t_{\text{iso}} = \frac{L^2}{\lambda v}. \quad (9)$$

The time for the density and pressure adjustments, on the other hand, is

$$t_{\text{mech}} = \frac{L}{c_s} \simeq \frac{L}{v}. \quad (10)$$

The ratio of the two times,

$$\frac{t_{\text{iso}}}{t_{\text{mech}}} \simeq \frac{L}{\lambda} \quad (11)$$

would have been large, if λ were short. However, in the neck region $\rho \sim \rho_0/3$ and thus $\lambda \sim 6$ fm. At the same time the distance from the nuclei to the center of the neck is $L \sim 5$ fm and we are in the Knudsen limit! There is no difference between the times for isospin and density adjustments, both are of the order of 30 fm. Even for a Knudsen gas, the equilibration could take place through a contact with a thermostat, i.e. nuclei, but that does not happen in the simulation. We find that we are *not* dealing with a phase transition near equilibrium.

4 Model for the Transition to Quark-Gluon Plasma

Let us now turn to the other important phase transition in nuclear physics, to QGP. The phase transition is approached when hadrons increase in number, pushing out from their region the nonperturbative vacuum. This can make the hadrons lighter as the condensates responsible for hadronic masses are removed. The transition is expected to occur when hadrons completely overlap. The lattice calculations for baryonless matter point to a phase transition at the temperature of $T_c \sim 170$ MeV corresponding to the pion density

$\rho_\pi \simeq 0.5 \left(\frac{T_c}{\hbar}\right)^3 \sim 2\rho_0$. If the phase transition is associated with the hadrons filling up all space, the transition should be expected also in the relatively cold matter when hadron density is raised to comparable values due to compression, rather than thermal production. The idea is further illustrated with a simple quasiparticle model [8].

The model [8], based on the hadronic degrees of freedom, produces a decrease of masses in the vicinity of the phase transition, a large drop in masses across the transition, and a dramatic increase in the number of the degrees of freedom in the transition. At the same time, the properties of ground-state nuclear matter are correctly described. The model is specified by giving the energy density as a function of particle phase distributions in the form of kinetic part and interaction corrections depending on two densities, scalar and vector:

$$e = \sum_X \int d\mathbf{p} \epsilon_{\mathbf{p}}^X f^X(\mathbf{p}) + e_s(\rho_s) + e_v(\rho), \quad (12)$$

where

$$\rho_s = \sum_X \int d\mathbf{p} \frac{m^X m_0^X}{\sqrt{p^2 + m^{X2}}} f^X(\mathbf{p}), \quad (13)$$

and $\rho = \sum_X A^X \int d\mathbf{p} f^X(\mathbf{p})$. The scalar density counts all hadrons while the vector density is just the baryon density. As hadron density increases, the hadron masses drop, from (12)

$$\epsilon_{\mathbf{p}}^X = \frac{\delta e}{\delta f^X(\mathbf{p})} = \sqrt{p^2 + (S(\rho_s) m_0^X)^2} + A^X U(\rho), \quad (14)$$

by a common (for simplicity) factor S in our model. In a thermally excited system the lowering of the masses leads to the production of more hadrons. Instability can occur leading to a phase transition across which the masses drop and the number of the degrees of freedom rapidly increases. We adopt simple power parametrizations of the mass modification factor and of the vector potential to get consistency with lattice calculations and ground-state nuclear-matter properties,

$$S = (1 - 0.54 (\text{fm}^3/\text{GeV}) \rho_s)^2, \quad (15)$$

and

$$U = \frac{a (\rho/\rho_0)^2}{1 + b \rho/\rho_0 + c (\rho/\rho_0)^{5/3}} \quad (16)$$

where $a = 146.32$ MeV, $b = 0.4733$, $c = 51.48$. With the transition in the baryonless limit at $T_c = 170$ MeV, we find a corresponding transition at $T = 0$, taking the matter from the baryon density of $\rho \approx 3.5 \rho_0$ to $7 \rho_0$, cf. Fig. 5.

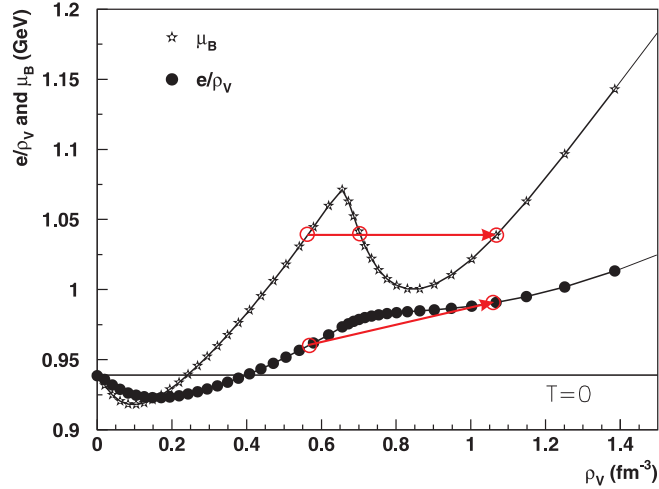


Figure 5: Chemical potential and energy per baryon at $T = 0$ as a function of baryon density. The high-density phase transition is indicated with arrows.

5 Elliptic-Flow Excitation Function

A possible way of detecting the high-energy nuclear phase transition is through the associated changes in the speed of sound. Thus, e.g. in the phase-transition region in Fig. 5, the speed of sound in the long wavelength limit ($c_s = \sqrt{\partial p / \partial e}$) vanishes.

A sensitive measure of the speed of sound or pressure compared to the energy density early on in the reactions is the elliptic flow. The elliptic flow is the anisotropy of transverse emission at midrapidity. At AGS energies, the elliptic flow results from a strong competition [9] between squeeze-out and in-plane flow. In the early stages of the collision, the spectator nucleons block the path of participant hadrons emitted toward the reaction plane; therefore the nuclear matter is initially squeezed out preferentially orthogonal to the reaction plane. In the later stages of the reaction, the geometry of the participant region (i.e. a larger surface area exposed in the direction of the reaction plane) favors in-plane preferential emission.

The squeeze-out contribution to the elliptic flow and the resulting net direction of the flow depend on two factors: (i) the pressure built up in the compression stage compared to the energy density, and (ii) the passage time for removal of the spectator shadowing. In the hydrodynamic limit, the characteristic time for the development of expansion perpendicular to the reaction plane is $\sim R/c_s$, where R is the nuclear radius. The passage time is $\sim 2R/(\gamma_0 v_0)$, where v_0 is the c.m. spectator velocity. The squeeze-out contribution should

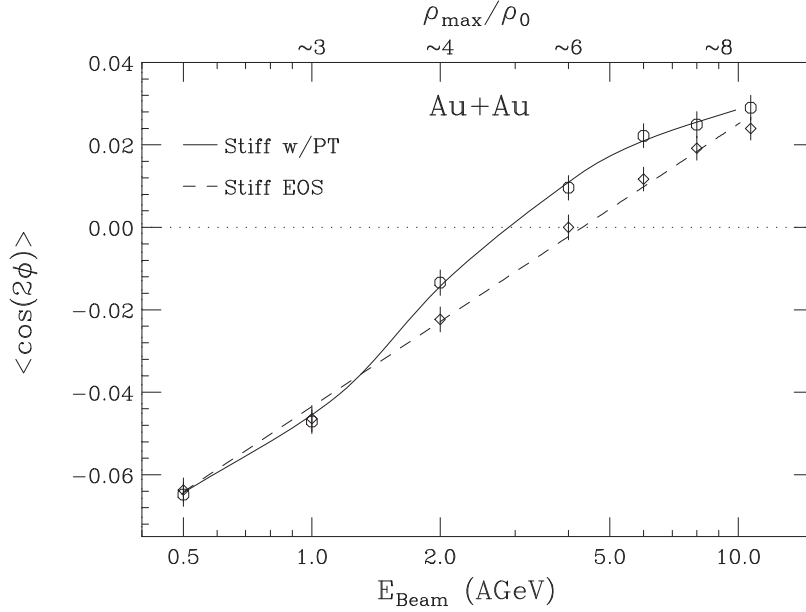


Figure 6: Calculated elliptic flow excitation functions for Au + Au. The open diamonds represent results obtained with a stiff EOS. The open circles represent results obtained with a stiff EOS and with a phase transition.

then reflect the ratio

$$\frac{c_s}{\gamma_0 v_0} . \quad (17)$$

According to (17) the squeeze-out contribution should drop with the increase in energy, because of the rise in v_0 and then in γ_0 . A stiffer equation of state (EOS) should yield a higher squeeze-out contribution. A rapid change in the stiffness with baryon density and/or excitation energy should be reflected in a rapid change in the elliptic flow excitation function. A convenient measure of the elliptic flow is the Fourier coefficient $\langle \cos 2\phi \rangle \equiv v_2$, where ϕ is the azimuthal angle of a baryon at midrapidity, relative to the reaction plane. When squeeze-out dominates, the Fourier coefficient is negative.

To verify whether the expectations regarding the elliptic-flow excitation function are realistic, we have carried out Au + Au reaction simulations [10], in the energy range of (0.5–11) GeV/nucleon. The excitation functions calculated using a stiff EOS with a phase transition (open circles) and a stiff EOS with a smooth density dependence are compared in Fig. 6. For low beam energies ($\lesssim 1$ AGeV), the elliptic flow excitation functions are essentially identical because the two EOS are either identical or not very different at the densities and temperatures that are reached. For $2 \lesssim E_{\text{Beam}} \lesssim 9$ AGeV

the excitation function shows larger in-plane elliptic flow from the calculation which includes the phase transition, indicating that a softening of the EOS has occurred for this beam energy range. This deviation is in direct contrast to the essentially logarithmic beam energy dependence obtained (for the same energy range) from the calculations which assume a stiff EOS without the phase transition. Present data on elliptic flow from EOS, E895, and E877 Collaborations [11] point to a variation in the stiffness of EOS in the region of $\sim(2-3)$ GeV/nucleon, corresponding to baryon densities of $\sim 4\rho_0$.

6 Conclusions

To summarize, the observed large enhancements in the yields of neutron-rich clusters in the neck region of reactions seem to be due to the deuteron and alpha production. Phase transition to QGP at high T in baryonless matter implies a transitional behavior at $T = 0$ and high baryon density. Elliptic flow measurements can decide about the presence or absence of the QGP transition with the rising density.

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